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Noise in a quantum point contact due to a fluctuating impurity configuration

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Abstract. We propose a theoretical model for the low-frequency noise observed in a quantum point contact (QPC) electrostatically defined in the 2D electron gas at a GaAs–AlGaAs interface. In such contacts electron scattering by soft impurity or boundary potentials coherently splits an incoming wave function between different transverse modes. Interference between these modes has been suggested to explain observed non-linearities in the QPC conductance. In this study we invoke the same mechanism and the time-dependent current due to soft dynamical impurity scattering in order to analyse the low-frequency (telegraph-like) noise which has been observed along with a non-linear conductance. For the simplified case of a channel with two extended (current carrying) modes, a simple analytical formula for the noise intensity is derived. Generally we have found qualitative similarities between the noise and the square of the transconductance. In comparison with the more traditional d.c. transport measurements we believe that noise measurements can provide additional information about the dynamical properties of QPCs.

1. Introduction

For several years problems related to electron transport through ballistic point contacts, shown in figure 1, have drawn a lot of attention from the solid state physics community. The most interesting feature of the so-called quantum point contacts (QPCs) is the non-linear character of their current–voltage (I–V) characteristics. This non-linearity is usually explained within the framework of conductance quantization [1, 2, 3, 4] (for a review see [5] and references therein). According to this concept the QPC forms a quantum channel that behaves as a wave guide for electrons, the number of transverse modes being dependent both on gate voltage V_g and source–drain voltage V_{sd}. Consequently, non-linear features of the I–V_{sd} curve should be observed at driving voltages of the order of the mean spacing between the quantized transverse energy levels; i.e. for $eV_{sd} \approx E_F/N$ (here E_F is the Fermi energy and N the number of modes). For typical parameters this corresponds to $V_{sd} \approx 1$ mV. Nevertheless, non-linear structure in the response has been observed [6, 7] at much lower voltages of order 0.01 mV. One could understand such a behaviour by taking into account a resonant structure in the conduction steps (see e.g. [8, 9, 10, 11]) due to scattering caused by abrupt variations in shape at the entrance and exit regions of the QPC. An explanation appropriate for QPCs with a smooth geometry, which probably corresponds to the experimental situation [6, 7], has been proposed in [12]. It is based upon the concept of *coherent mode mixing* inside the contact; an electron wave entering the

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contact is coherently split between different transverse modes due to a scattering potential. During propagation along the channel the electron wave packet becomes deformed because the phase shifts gained by different transverse mode components are not the same. As a result, an interference structure in the current appears because of mode mixing after a second scattering event. Such a behaviour is very similar to the well known electrostatic Aharonov–Bohm effect (see e.g. [5]). It is very important to notice that the described model does not rely upon a short-range scattering potential which can produce strong backscattering. On the contrary, the effect persists even if the scattering potential is soft, which is the case in ballistic structures [13, 14].

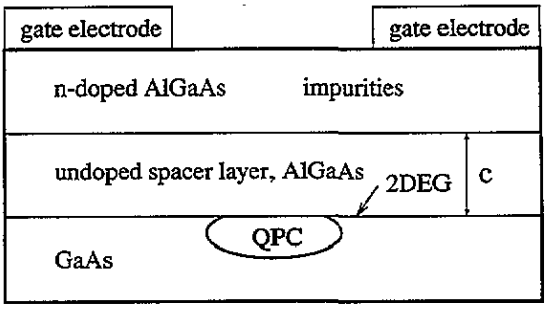

Figure 1. A quantum point contact is defined at the interface between the undoped AlGaAs spacer layer and GaAs by the split gate confinement technique. Noise is caused by the motion of impurities in the n-doped AlGaAs layer.

The model introduced in [12] explains the qualitative structure in the measured I–V curves as well as their main quantitative characteristics. We believe that the above-mentioned mechanism is responsible for the features observed in [6] and [7].

Along with a non-linear oscillating contribution to the I–V curve of QPCs, a telegraph-like low-frequency noise [15] has also been found. This kind of oscillating noise (as function of gate and source–drain voltages) is to be distinguished from the type of noise in [23] and [24]. In their case, noise is due to switching of the number of current carrying modes which in turn is caused by fluctuations of the Fermi level. Our aim here is to analyse the former type of oscillating noise in a QPC within the framework of the model introduced in [12]. Comparison of results on telegraph noise with available experimental data could support (or falsify) the above-mentioned model.

The paper is organized as follows. In section 2 the model employed will be discussed. General analytical expressions for telegraph-like noise, as well as its relation to current, will be given in section 3. In the last section 4 we discuss main results and present our conclusions.

2. Model

Following [12], we model our QPC—formed in a 2D electron gas by gate electrodes—as an adiabatically smooth channel [16] connecting two equilibrium reservoirs. We assume that the QPC contains two scatterers. The origin of these scattering centres, both taken to be static for the moment, can be the soft potentials formed by one or several of the impurity atoms in the n-doped AlGaAs layer (see figure 1) that are normally present because of doping. The gate electrodes are assumed to provide a hard-wall confining potential in the transverse direction, hence creating a channel whose width is furthermore assumed to vary

smoothly in the longitudinal direction. The WKB approximation for the electron wave function is therefore applicable. One finds [16]

$$\Psi(x, y, E) = \sum_{n,\pm} a_n^{\pm} \chi_n^{\pm}(x, y, E) \tag{1}$$

where

$$\chi_n^{\pm}(x, y, E) = \sqrt{k_{n,\parallel}(E, \mp\infty)/k_{n,\parallel}(E, x)}\,\phi_{n,x}(y) \exp\left[\mathrm{i}\int_{\mp\infty}^{x} \mathrm{d}x'\, k_{n,\parallel}(E, x')\right] \tag{2}$$

$$k_{n,\parallel}(E, x) = k_F\sqrt{\varepsilon - \varepsilon_{n,\perp}(x) - ug(x) - vs(x)}. \tag{3}$$

Here $E \equiv \varepsilon E_F$ is the total energy of the electron—E_F being the Fermi energy in the leads at zero bias voltage, $V_{sd} = 0$—while $k_F = \left(2mE_F/\hbar^2\right)^{1/2}$ is the Fermi wave vector and $k_{n,\parallel}(E, x)$ the longitudinal wave vector along the channel. The transverse part of the wave function, $\phi_n(y)$, depends parametrically on the longitudinal coordinate x; the corresponding 'transverse' energy eigenvalue is $\varepsilon_{n,\perp}$ and is measured in units of the Fermi energy E_F. Hence

$$\varepsilon_{n,\perp} = \left(\frac{\pi n}{k_F d(x)}\right)^2 \tag{4}$$

where $d(x)$ is the coordinate-dependent width of the channel. The distribution of the electrostatic potential caused by the applied source–drain voltage is affected by all charges within the contact region; it is described by the dimensionless parameters $v = eV_{sd}/2E_F$ and $s(x)$ which appear in the combination $vs(x)$ in (3). In order to match the Fermi levels in the leads we must obviously require that $s(\pm\infty) = \pm 1$. The distribution along the channel for the gate voltage modelled by the dimensionless parameter u is described by the dimensionless function $g(x)$. The plus sign (+) in (2) corresponds to transmission from the left to the right reservoir, while a minus sign (−) corresponds to transmission in the opposite direction. Without losing anything essential, we simplify our model by letting $g(x) = 1$, $v = 0$ in expression (3) for the WKB wave functions.

The interference effects of interest to us are due to coherent scattering by two scatterers. The first one splits an incoming mode into several other modes which then propagate independently. Making use of the unitarity of the scattering matrix one can show [12, 17] that such a splitting does not change the current at all in the absence of backscattering (if the potential is soft enough). The unitarity condition is simply a statement of conservation of probability: total incoming flux must equal total outgoing flux. The second scatterer makes an additional coherent mode mixing that leads to an interference pattern in the total transmission (or reflection) coefficient, and in the current.

Let us assume that an electron enters from the left, is transmitted after scattering against the left impurity (L), propagates through the contact, passes the right impurity (R) after a second scattering event, and finally escapes the channel on the right-hand side. To describe the coherent splitting of the WKB wave function (1) we introduce—following [12]—for each scatterer a unitary $(2N \times 2N)$ scattering matrix,

$$\hat{S} \equiv \begin{pmatrix} \hat{r}^- & \hat{t}^+ \\ \hat{t}^- & \hat{r}^+ \end{pmatrix} \tag{5}$$

such that

$$\begin{pmatrix} \Psi_{\text{out}}^+ \\ \Psi_{\text{out}}^- \end{pmatrix} = \hat{S} \begin{pmatrix} \Psi_{\text{in}}^- \\ \Psi_{\text{in}}^+ \end{pmatrix}. \tag{6}$$

The $(N \times N)$ submatrices $\hat{r}^{\pm}$ and $\hat{t}^{\pm}$ define the reflection from, and the transmission through, the scatterer of the components of the incident wave function. The plus/minus component $\Psi_{\text{in}}^{\pm}$ refers to the incident wave coming from the left/right reservoir, while the plus/minus component $\Psi_{\text{out}}^{\pm}$ refers to the outgoing wave from the scatterer propagating to the right/left. The electron propagation right/left between the scattering events is described by diagonal *phase gain matrices* $\hat{U}$,

$$\hat{U}_{ij}^{\pm} = \delta_{ij} \exp\left[\pm i\sigma_j(E)\right] \tag{7}$$

where

$$\sigma_j(E) = \int_{x_L}^{x_R} \mathrm{d}x' \, k_{j,\parallel}(E, x') \tag{8}$$

is the phase gained by the jth mode between scatterers R and L.

The general expression for the current is [12]

$$I(V_{sd}) = \frac{2e}{h} \int \mathrm{d}E \, \delta n_{\text{F}}(E) \, \mathrm{Tr}\left[\hat{T}^{+}(E)^{\dagger}\hat{T}^{+}(E)\right]$$
$$\delta n_{\text{F}}(E) \equiv n_{\text{F}}(E - E_{\text{F}} - eV_{sd}/2) - n_{\text{F}}(E - E_{\text{F}} + eV_{sd}/2) \tag{9}$$

where δn_{F} is the difference between the Fermi–Dirac distribution functions in the reservoirs, and $\hat{T}^{+}(E)$ the total transfer matrix (for transfer from left to right) for a particle with energy E, so that $\Psi_{\text{out}}^{+} = \hat{T}^{+}\Psi_{\text{in}}^{+}$. To lowest order, $\hat{T}^{+} = \hat{t}_R^{+}U^{+}\hat{t}_L^{+}$. Making use of (5) and (7), one can show [12] that the total current can be expressed as a sum of two parts: $I = I_{\text{diag}} + I_{\text{int}}$. The first part

$$I_{\text{diag}} = \frac{2e}{h} \int \mathrm{d}E \, \delta n_{\text{F}}(E) \sum_j R_{jj}(E) L_{jj}(E) \tag{10}$$

originates from diagonal parts of the matrices $\hat{R} \equiv \hat{t}_R^{+\dagger}\hat{t}_R^{+}$ and $\hat{L} \equiv \hat{t}_L^{+}\hat{t}_L^{+\dagger}$. The second contribution,

$$I_{\text{int}} = \frac{2e}{h} \int \mathrm{d}E \, \delta n_{\text{F}}(E) \sum_{j<k} 2 \, \mathrm{Re}\left(R_{jk}L_{kj} \, \mathrm{e}^{i\sigma_{jk}}\right) \tag{11}$$

where $\sigma_{jk} \equiv \sigma_j - \sigma_k$, contains all interference effects. In the absence of backscattering (10) reduces to the usual expression for the current through a ballistic constriction, which exhibits the well known conductance quantization with gate voltage [1]. I_{int} is the contribution from interference between different modes. The expressions (10) and (11) that give the total current were analysed in [12]. They are the starting points for calculating low-frequency noise.

3. Low-frequency noise

3.1. General consideration

One can imagine several mechanisms which lead to low-frequency noise—external, such as low-frequency fluctuations of gate and source–drain voltages, and internal, such as spatial rearrangements of the scattering potentials. In the present study we consider the latter mechanism, which we believe to be the simplest one.

It is well known that there is some disorder in the vicinity of small devices, even if they are of high quality. In any disordered system defects with internal degrees of freedom are present. Interactions with a thermal bath can induce transitions between the corresponding quantum mechanical states (see [18] for a review). Usually, such defects switch between two

states, leading to a telegraph-like noise pattern. Dynamical defects of this kind have been observed in metal point contacts and tunnel junctions by several authors (see, e.g. [25–30] and have been called 'elementary fluctuators', or EFs for brevity. Typically, a Lorentzian frequency dependence of noise is observed if one EF is responsible for the fluctuations, while in the case of several activated EFs usually a $1/f$ dependence (often called flicker noise) is found [24–30]. The number of activated EFs increases with temperature, so one can actually in some cases see the crossover from Lorentzian to $1/f$-like noise when the temperature is raised [24]. In the present study we focus on noise from a single EF. Thus, for our results to be applicable, bandwidth-limited noise should be observed.

The microscopic structure of the EFs is not completely clear. One of the possible sources of two-state defects is disorder-induced soft atomic vibrations. For low excitation energies the vibrations are strongly anharmonic and can be described as an atom or group of atoms moving in an effective double-well potential. Such entities are known as [31, 32] two-level tunnelling systems (TLS). They are responsible for the low-temperature properties of glassy materials. The generalization of the TLS model for higher excitation energies has been worked out in [33]. Dynamical defects produce elastic (or electric) fields, slowly varying in time. Conduction electrons are then scattered by these fields. Another possible origin of fluctuations in the scattering potential is electron hopping between adjacent sites in the doped region of the device. The motion of the EFs leads to a variation of the scattering potential in the 2DEG region.

A quantitative theory is not yet completely worked out for either of the two mechanisms mentioned above. In order to describe the main physical picture we use a simplified model: i.e. we assume that one of the scatterers (the left one, say) will maintain its static character, while the other one (right) is allowed to hop between two different spatial positions. The hopping scatterer will from now on be denoted the elementary fluctuator (EF). Its spatial position x_R is a random quantity, which we describe as

$$x_R(t) = x_R^0 - \frac{l}{2}\xi(t). \tag{12}$$

Here l is the hopping distance; for simplicity, the transverse coordinate has been neglected, leading to an effectively one-dimensional description. The random quantity $\xi(t)$ is jumping between the two values ± 1 at random times, thus describing a telegraph-like process.

The hopping is induced by an interaction with a thermal bath. The transition rate Γ_+ from the state $\xi = +1$ to $\xi = -1$ and the reverse rate Γ_- are determined by the nature of hopping and by the interaction between the EF and the thermal bath. From the detailed balance principle we have

$$\Gamma_-/\Gamma_+ = \exp(-\Delta/k_B T) \tag{13}$$

where T is the temperature, and Δ the energy difference between the states of the EF. Consequently, at high enough temperatures ($k_B T \gg \Delta$) the hopping rates are almost equal, while at low temperatures there will be a significant difference between the two. The dependence of $\Gamma = \Gamma_+ + \Gamma_-$ on Δ and T is determined by the hopping mechanism (see the discussion in [18] and [24]). If the transitions are due to quantum mechanical tunnelling, $\Gamma \propto \Delta^k$ where $k = 3$ for EF–phonon interaction and $k = 1$ for EF–electron interaction. If transitions are induced by activation, on the other hand, $\Gamma \propto \exp(-W/k_B T)$ where W is some activation energy [18]. We assume the EF to be unaffected by the applied source–drain voltage as well as the gate voltage [19]. Thus our model does not apply to the mechanically controllable break junctions studied in [21]. For normal semiconductor junctions though, the electric fields generated by these voltages in a remote EF (see figure 1) are usually much less than the EF's internal field. Hence Γ can be considered to be independent of

these voltages [20]. We assume that the only effect of the hops made by the scatterer is a variation in the phases, σ_j, gained by conducting modes. As has been shown in [12] the oscillating part of the current is sensitive to the phase of the modes. Therefore, we expect that the simple model we use contains the most important mechanism for the influence on the current from the EF.

3.2. Analytical expression for noise intensity

Noise is usually characterized by the current–current correlation function

$$S(\tau) = \langle I(t+\tau)I(t)\rangle_t - \langle I(t)\rangle_t^2 \tag{14}$$

or by its Fourier transform $S(\omega)$ with respect to time τ. The symbol $\langle\cdots\rangle_t$ means average over time t which is (under stationary conditions) just the same as an average over the random process $\xi(t)$ (ergodicity). Strictly speaking, the current I is an operator, and the procedure also has to include a quantum mechanical average. However, here we are interested in long-time correlations while the so-called quantum noise [34]

$$\langle\hat{I}(t+\tau)\hat{I}(t)\rangle_q - \langle\hat{I}(t+\tau)\rangle_q\langle\hat{I}(t)\rangle_q$$

(subscript q stands for a quantum mechanical average) decays at very short times τ. Consequently, to study flicker noise one can replace current operators by their quantum mechanical averages.

Because the diagonal part of the current is time independent, only the interference part enters the expression for noise. Making use of equation (11) for $I_{\rm int}$ we obtain the following form of the current–current correlation function:

$$S(\tau) = \left(\frac{2e}{h}\right)^2 \int \mathrm{d}E\,\delta n_{\rm F}(E) \int \mathrm{d}E'\,\delta n_{\rm F}(E') \sum_{j\neq k,l\neq m} M_{jklm}(E,E')\Phi_{jklm} \tag{15}$$

where

$$\begin{aligned} M_{jklm} &= R(E)_{jk}L(E)_{kj}R(E')_{lm}L(E')_{ml} \\ \Phi_{jklm} &= \langle \mathrm{e}^{\mathrm{i}\sigma_{lm}(t,E')+\mathrm{i}\sigma_{jk}(t+\tau,E)}\rangle_t - \langle \mathrm{e}^{\mathrm{i}\sigma_{lm}(t,E')}\rangle_t\langle \mathrm{e}^{\mathrm{i}\sigma_{jk}(t,E)}\rangle_t. \end{aligned} \tag{16}$$

Assuming that the longitudinal wave vector $k_{j,\parallel}$ varies slowly as a function of position x_R in the channel, one can approximate the phase as

$$\begin{aligned} \sigma_j(t,E) &\approx \sigma_j^0(E) + w_j(E)\xi(t) \\ w_j(E) &\equiv k_{j,\parallel}(E,x_R^0)l/2. \end{aligned} \tag{17}$$

The superscript 0 indicates that the phase, (8), should be evaluated for x_R^0. We then arrive at the following approximate form of the function (16):

$$\Phi_{jklm} \approx \mathrm{e}^{\mathrm{i}[\sigma_{jk}^0(E)+\sigma_{lm}^0(E')]}\mathcal{G}\left[w_{jk}(E), w_{lm}(E')|\tau\right] \tag{18}$$

where $w_{jl}(E) \equiv w_j(E) - w_i(E)$, and

$$\begin{aligned} \mathcal{G}(x,y|\tau) &= K(x,y|\tau) - K(x,y|\infty) \\ K(x,y|\tau) &= \left\langle \mathrm{e}^{\mathrm{i}x\xi(t+\tau)+\mathrm{i}y\xi(t)}\right\rangle_t. \end{aligned} \tag{19}$$

Note that the function $K(x,y|\tau)$ is known [36] as the *generating function* for the random process $\xi(t)$. After a straightforward analysis, we get from equation (A7) (see appendix A)

$$\mathcal{G}(x,y|\tau) = -\sin x \sin y \frac{4\Gamma_+\Gamma_-}{\Gamma^2}\mathrm{e}^{-\Gamma|\tau|} = -\frac{\sin x \sin y}{\cosh^2(\Delta/2k_{\rm B}T)}\mathrm{e}^{-\Gamma|\tau|}. \tag{20}$$

Thus we find that the quantity $\mathcal{G}\left[w_{jk}(E), w_{lm}(E')|\tau\right]$ in equation (18) can be factorized into functions of E and E'. Furthermore, by their mere construction the matrices $\hat{R}$ and $\hat{L}$ are Hermitian. Hence one can reduce the double sum as follows:

$$\sum_{j\neq k} \sin w_{jk}(E) R(E)_{j,k} L(E)_{kj}\, \mathrm{e}^{\mathrm{i}\sigma^0_{jk}(E)}$$
$$= 2\mathrm{i} \sum_{j<k} \sin w_{jk}(E)\, \mathrm{Im}\left[R_{jk}(E) L_{kj}(E) \mathrm{e}^{\mathrm{i}\sigma^0_{jk}(E)}\right]. \tag{21}$$

Taking the Fourier transform with respect to τ we find the noise spectrum to be a Lorentzian since the current–current correlation function is exponentially decreasing in time.

Collecting the partial results above, we obtain a general expression for the noise of the form

$$S(\omega) = \frac{1}{\cosh^2(\Delta/2k_\mathrm{B}T)} \mathcal{L}(\omega) \Lambda^2$$
$$\mathcal{L}(\omega) = \frac{1}{\pi} \frac{\Gamma}{\Gamma^2 + \omega^2} \tag{22}$$

where

$$\Lambda = \frac{2e}{\hbar} \int \mathrm{d}E\, \delta n_\mathrm{F}(E) \sum_{j<k} \sin w_{jk}(E, x^0_R)\, 2\, \mathrm{Im}\left[R_{jk}(E) L_{kj}(E) \mathrm{e}^{\mathrm{i}\sigma^0_{j,k}(E)}\right]. \tag{23}$$

The expression (22) is a product of three factors. The first one, $\cosh^{-2}(\Delta/2k_\mathrm{B}T)$, makes it clear that a telegraph-like noise can only appear if the EF can hop due to interactions with a thermal bath. The typical excitation energy is of order $k_\mathrm{B}T$. If $\Delta \gg k_\mathrm{B}T$ the EF cannot be excited and will remain in its lowest-energy state forever. The second factor is a simple Lorentzian function with characteristic width Γ. The third factor in (22), Λ^2, is equal to $(I^+_\mathrm{int} - I^-_\mathrm{int})^2/4$ where superscripts $\pm$ correspond to the values ± 1 of the random process $\xi(t)$. As a result, equation (22) can be expressed as

$$S(\omega) = \frac{\Gamma_+\Gamma_-}{\Gamma^2} \left[I^+_\mathrm{int} - I^-_\mathrm{int}\right]^2 \mathcal{L}(\omega). \tag{24}$$

For the particular case when there are only two possible values of the interference current one can derive this expression directly from the Master equation without using the generating function. We still keep the above derivation though because it allows for a straightforward generalization to the case of several EFs.

The quantity Λ contains the same parameters as expression (11) for the interference part of the current. It is therefore of interest to see whether or not the quantity Λ can be related to measured (average) current–voltage characteristics. This is the subject of the next section.

3.3. Noise and interference current

Since the interference part of the current (11) is fluctuating, the average current will be the most easily measured quantity. Provided $R_{jk}L_{kj}$ is real, we only have to calculate $\mathrm{Re}\langle \mathrm{e}^{\mathrm{i}\sigma_{jk}(t)}\rangle$ to find this average. Using the approximation for the phase (17), and the definition of $K(x, y|\tau)$ in (19), we may express the average of the phase factor in terms of K as follows:

$$\langle \mathrm{e}^{\mathrm{i}\sigma_{jk}(t)}\rangle_t = \mathrm{e}^{\mathrm{i}\sigma^0_{jk}} \langle \mathrm{e}^{\mathrm{i}w_{jk}\xi(t)}\rangle_t = \mathrm{e}^{\mathrm{i}\sigma^0_{jk}} K(w_{jk}, 0|0). \tag{25}$$

From (A6) (see appendix A) and (13) we then immediately obtain the average current,

$$\langle I_{\rm int}(t)\rangle_t = \frac{2e}{h}\int {\rm d}E\,\delta n_{\rm F}(E)\sum_{j<k} 2R_{jk}L_{kj}(E)\left[\cos(\sigma^0_{jk}(E))\cos(w_{jk}(E))\right.$$
$$\left. + \tanh(\Delta/2k_{\rm B}T)\sin(\sigma^0_{jk}(E))\sin(w_{jk}(E))\right]. \tag{26}$$

Besides the current often the transconductance, here defined as

$$G(V_{sd}, V_g) = \frac{\partial\langle I_{\rm int}(t)\rangle_t}{\partial V_g} \tag{27}$$

is measured. Only the interference part of the current has been included because far from the conduction steps the diagonal part is practically independent of the gate voltage.

Further, from now on we restrict ourselves to only consider straight channels. This simplification allows for a simple estimate of the phase differences $w_{jk}(E_{\rm F})$ and $\sigma^0_{jk}(E_{\rm F})$ ($\hat{k}_{j,\parallel} \equiv k_{j,\parallel}(u=0)/k_{\rm F}$),

$$\begin{pmatrix} w_{jk} \\ \sigma^0_{jk} \end{pmatrix}(u) = \left(1 + \frac{u}{2\hat{k}_{j,\parallel}\hat{k}_{k,\parallel}}\right)(\hat{k}_{j,\parallel} - \hat{k}_{k,\parallel})\begin{pmatrix} k_{\rm F}l/2 \\ k_{\rm F}L \end{pmatrix} \leqslant \mathcal{O}\begin{pmatrix} k_{\rm F}l/2 \\ k_{\rm F}L \end{pmatrix} \tag{28}$$

provided we are far from the thresholds where the number of conducting modes changes. Remember that $L = x_L - x^0_R$ is the distance between the scatterers and l the hopping distance of the EF. The period of oscillation in noise as a function of the dimensionless gate voltage u will be

$$\delta u = \frac{2\pi}{k_{\rm F}L}\frac{\hat{k}_{j,\parallel}\hat{k}_{k,\parallel}}{\hat{k}_{j,\parallel} - \hat{k}_{k,\parallel}} \tag{29}$$

Since typically $k_{\rm F}l \leqslant 1$, we find it sufficient to expand all results to lowest order in w_{jk}. This upper bound of $k_{\rm F}l$ calls for a digression. The typical displacement l of an EF in the course of thermal-bath-induced fluctuations depends strongly on the microscopic nature of the EF. If the EF is produced by structural disorder in the vicinity of the channel, one can expect l to be of the order of an interatomic distance, and $k_{\rm F}l \ll 1$. If the EF originates from electron hopping in the doped region of the structure, it is reasonable to expect that l is of the order of the average distance between the impurities in the doped region. In this case the product $k_{\rm F}l$ can be as large as 1–10 depending on size and shape of the structure. Another point to note is that a mechanical displacement of the scatterer is not the only reason for variations in the scattering phase $k_{\rm F}l$. Rearrangements in an extended defect containing several atoms can also cause variations in the scattering phase of order unity. Our phenomenological model does not allow for explicit calculations of the scattering phase, so we keep the simplest description of the mechanical displacement of the scatterer and consider the product $k_{\rm F}l$ to be of order one.

3.3.1. Two propagating modes. To compare noise and current, we first consider a simple case for which we do not explicitly have to calculate $R_{jk}L_{kj}$: when the QPC only allows for two propagating modes, noise can for low temperatures ($T \ll \Delta/k_B, T_{\rm F}$) and low source–drain voltages V_{sd} be directly related to the current,

$$\langle I_{\rm int}(t)\rangle_t = I_{\rm max}\cos\left[\sigma^0_{12}(E_{\rm F}) - w_{jk}(E)\right] \tag{30}$$

$$G(u) = \frac{I_{\rm max}k_{\rm F}L}{2E_{\rm F}}\left[\frac{k_{\rm F}}{k_{1,\parallel}} - \frac{k_{\rm F}}{k_{2,\parallel}}\right]\sin\sigma^0_{12}(E) \tag{31}$$

$$S(\omega) = I_{\max}^2 \frac{w_{12}^2(E_F)\sin^2\left[\sigma_{12}^0(E_F)\right]}{\cosh^2(\Delta/2k_BT)}\mathcal{L}(\omega) \tag{32}$$

$$I_{\max} = 2g_0 V_{sd} R_{12}(E_F) L_{21}(E_F) \tag{33}$$

where $g_0 = 2e^2/h$ is the unit of conductance. $I_{\max}$ is the maximum value of the interference current with respect to variations in external parameters.

Qualitative conclusions about expressions (30)–(33) will be drawn in section 4.

3.3.2. Many propagating modes. In the case when there are more than two modes propagating through the quantum point contact, one has to solve for the product $R_{jk}(E)L_{kj}(E)$ of the scattering rates between different modes. Generally the solution to the scattering problem is rather involved and since there are many unknown parameters we will in this section only make estimates of the order of magnitude of these scattering rates. It will turn out that the largest contribution, within our approximations, comes from scattering between the propagating modes with the two lowest mode indices. Thus we can actually apply the two-mode results (30)–(33) in section (3.3.1) even when there are more than two current carrying modes. However, the approximations adopted below are less accurate the more modes there are, see numerical estimates in table 1.

Table 1. Square of intermode scattering rates, $T_{jk}(E_F)$, between modes j and k at the Fermi energy and for zero gate voltage. The parameter N is the total number of propagating modes and the channel width was chosen so as to have the contact biased at the centre of a plateau in conductance quantization. To have a nonzero value of T_{jk} we must have odd values of $j \pm k$. It is clearly seen that increasing the total number of modes N and/or decreasing k_Fa and k_Fb makes the differences smaller.

N	k_Fa	k_Fb	Mode j	Mode k	$T_{jk}(E_F)/T_{12}(E_F)$
5	10	20	1	2	1
			2	3	2×10^{-2}
			3	4	8×10^{-3}
			4	5	3×10^{-5}
			1	4	4×10^{-7}
			2	5	8×10^{-13}
	20	40	1	2	1
			2	3	3×10^{-3}
			3	4	9×10^{-5}
			4	5	2×10^{-9}
			1	4	1×10^{-11}
			2	5	2×10^{-21}
10	10	20	1	2	1
			2	3	6×10^{-2}
			3	4	1×10^{-1}
			4	5	3×10^{-2}
			5	6	4×10^{-2}
			6	7	1×10^{-2}
			7	8	8×10^{-3}
			8	9	1×10^{-3}
			9	10	1×10^{-4}
			1	4	9×10^{-4}
			2	5	4×10^{-6}
			3	6	2×10^{-5}
			etc		

The quantity $R_{jk}(E)L_{kj}(E)$ is the product of the two scattering probabilities for scattering from mode j to k by the left scatterer and the analogous probability—but time reversed and with exchanged mode indices—for the right scatterer. Since we have restricted our channel to be straight, $R_{jk}(E) = L_{jk}(E)$. To simplify the calculations we assume that the only screening of the impurity potential that exists originates from electrons within the gate electrode. This screening can approximately be described by a proper image charge. If the impurity and image charges are separated from the 2DEG by distances a and b, respectively, the effective potential is

$$V(r) = A_0\left[\frac{1}{\sqrt{(r^2+a^2)}} - \frac{1}{\sqrt{(r^2+b^2)}}\right] \tag{34}$$

where r is the in-plane coordinate, while A_0 contains physical constants.

The scattering probability can be calculated from the Golden Rule,

$$L_{kj}(E) = \frac{2\pi}{\hbar}|\langle f|V(x,y)|i\rangle|^2\,\delta(E_f - E_i) \tag{35}$$

where i and f denote the initial and final states.

For $r \gg \max(a,b)$, we find that $V(r) \propto 1/r^3$. The potential (34) is hence short ranged, and its influence may be ignored for large r. In the opposite limit of small values of r, we get $V(r) = A_0(1/a - 1/b)[1-(r/c)^2]$, where $c = ab/\sqrt{(a^2+ab+b^2)/2}$. The typical range of $V(r)$ is c.

Due to the presence of the spacer layer (see figure 1), the width of the channel will most likely be much smaller than the distances a and b. In this case the impurity potential can be considered as being constant in the transverse direction. The matrix element in (35) will then simply be proportional to the overlap between the transverse parts of the wave functions. This overlap is non-zero only between modes of different parity (i.e. when $k \pm j$ is an odd number). The matrix element can consequently be written as

$$|\langle k|V(x,y)|j\rangle| = \frac{4}{\pi}\Omega(j+k)\frac{j\Omega(k)+k\Omega(j)}{|j^2-k^2|}|V_x(k_{k,\parallel}-k_{j,\parallel})| \tag{36}$$

where $\Omega(n) \equiv [1-(-1)^n]/2$ equals zero for even n and one for odd n. The function $V_x(q)$ is the Fourier transform of the potential along the channel. To evaluate $V_x(q)$ we must solve the integral

$$\begin{aligned}\int_{-\infty}^{\infty} dx\,\frac{e^{-iqx}}{\sqrt{x^2+a^2}} &= 2e^{-qa}\int_0^{\infty} dt\,\frac{e^{-qt}}{\sqrt{t(t+2a)}} \\ &\approx \sqrt{\frac{2}{qa}}e^{-qa}\int_0^{\infty} ds\,\frac{e^{-s}}{\sqrt{s}} \qquad \text{if } 2qa \gg 1.\end{aligned} \tag{37}$$

Here we have introduced branch cuts at $-i\infty \to -ia$ and at $ia \to i\infty$ and transformed the integral by means of complex contour integration. The distances a and b are large due to the presumed low intermode scattering; in the last step we thus are allowed to assume $k_F a, k_F b \gg 1$. A lower bound of these parameters is given by $k_F c$, where c is the spacer layer thickness (see figure 1). It can be estimated from [5] and [23] $k_F = 1.6 \times 10^6$ cm^{-1} and $c = 42$ nm implies $k_F c = 6.7$. The reason for having a spacer in the structure (see figure 1) is in fact to reduce impurity scattering so the scattering must be very soft. Indeed, in [7] the off-diagonal contribution (in our model due to intermode scattering) to the current was found to be a fraction $O(10^{-3})$ of the diagonal part of the current.

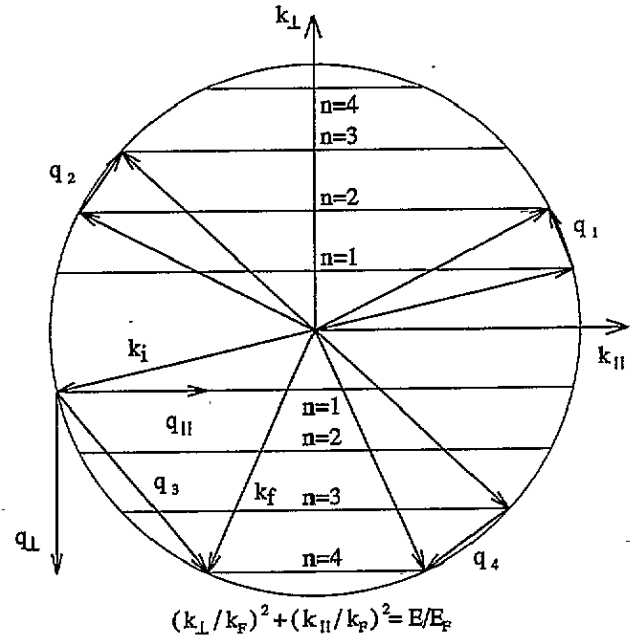

Figure 2. Illustration (taken from [22] and [23]) of possible scattering events for the case of four propagating modes far from threshold where the number of propagating modes changes. The signs of $k_\perp$, $k_\parallel$ and $q_\parallel$ are chosen for convenience and clarity. Because of the exponential dependence on $q_\parallel a$ and $q_\parallel b$ and the high value of $k_F a$ and $k_F a$, scattering case 1 gives a much larger contribution to T_{jk} than cases 2, 3 and 4. (See also table 1.)

The energy-dependent quantity determining the strength of the different intermode scattering rates will be

$$R_{jk}L_{kj}(E) \propto \Omega(j+k)\left[\frac{j\Omega(k)+k\Omega(j)}{j^2-k^2}\right]^4\left[\frac{e^{-q_\parallel a}}{\sqrt{q_\parallel a}}-\frac{e^{-q_\parallel b}}{\sqrt{q_\parallel b}}\right]^4\frac{k_F^2}{k_{j,\parallel}k_{k,\parallel}} \equiv T_{jk}(E). \qquad (38)$$

Of crucial importance here is the *exponential* dependence on the longitudinal component of the transferred momentum, $q_\parallel \equiv |k_{m,\parallel} - k_{j,\parallel}|$. This result agrees with findings by Price [35]. Along with the fact that $k_F a, k_F b \gg 1$, it implies that the major contribution comes from the scattering event with the lowest value of $q_\parallel$. If the total number of propagating modes is not very high, we see from figure 2 that the scattering from mode number one into mode number two gives the lowest value of $q_\parallel$. As an approximation we may put all other scattering amplitudes to zero. Hence, even in the more general case of more than two modes we should observe a similar relation between noise and current as in the two-mode case (see section 3.3.1) above. For completeness, numerical values of $T_{jk}(E_F)/T_{12}(E_F)$ are given in table 1 for different values of (j, k) and for a particular set of physical parameters.

4. Discussion and conclusions

Let us start the discussion by reviewing the results for a QPC with two current carrying modes, equations (30)–(33). In this case we predict that the period of noise oscillations as a function of external parameters should be half the period of oscillations in the current. Verification of this statement is important in order to confirm (or falsify) our model.

In the general case when there is no restriction on the number of conducting modes we conclude the following.

(i) By qualitatively estimating the intermode scattering rates we found that the major contribution to noise and interference current comes from scattering between modes one and two. However, decreasing the distance to the impurities and/or increasing the number of conducting modes makes this conclusion less accurate. The many-mode solution could thus be mapped on the result for two conducting modes. In particular, noise and square of transconductance should oscillate with the same frequency and phase, as functions of gate voltage.

(ii) The dependence on temperature of the telegraph noise is determined by the factor

$$\cosh^{-2}(\Delta/k_{\mathrm{B}}T).$$

Comparing with experiments one can determine the interlevel spacing, Δ, of the elementary fluctuator (EF). This would help to identify the nature of the EF.

(iii) The width, Γ, of the Lorentzian that describes the frequency dependence of the noise is also a function of temperature. In fact it reflects the most important dynamical property of the EF because the temperature dependence is different for quantum mechanical tunnelling and activation (see section 3.1).

(iv) The main feature of our model is that variations in external parameters (such as V_g, V_{sd}) are assumed to only affect the phase functions $\sigma_j(E)$ (see equation (8)). The oscillatory behaviour of the noise and the square of derivatives of the current (with respect to any external parameter) should then be qualitatively similar.

(v) The assumed nature of the EF causes the additional phase w_{ij} ($\propto l$) due to its fluctuating position to be small. To lowest order, noise will have a quadratic dependence on the hopping distance l. Thus if there are several EFs, the EF with the largest hopping distance will give the largest contribution to noise.

Finally, we would like to stress that, in spite of the idealized character of the adopted model, our conclusions seem rather general. Indeed, any kind of time fluctuation in the scattering phase shift by any scatterer should lead to a similar behaviour. In that case the parameter w_{ij} will have a different meaning and could in principle be of order unity.

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Appendix. Derivation of the generating function $K(x, y|\tau)$

In order to evaluate $K(x, y|\tau)$ let us analyse the Master equation for the conditional probability $Q(\xi', t'|\xi, t)$ of finding the value $\xi = \xi'$ at the time t' under the condition $\xi(t) = \xi$. It reads (cf. [36])

$$\frac{\partial Q(\xi', t'|\xi, t)}{\partial t} - \xi'\Gamma_+ Q(+1, t'|\xi, t) + \xi'\Gamma_- Q(-1, t'|\xi, t) = 0 \qquad \text{(A1)}$$

with initial condition $Q(\xi', t|\xi, t) = \delta_{\xi',\xi}$. Taking into account the sum rule

$$Q(+1, t'|\xi, t) + Q(-1, t'|\xi, t) = 1$$

we find the following solution to (A1):

$$Q(\xi', t'|\xi, t) = \frac{1}{2\Gamma}\left[\Gamma - \xi'(\Gamma_+ - \Gamma_-) + \xi' e^{-\Gamma|t'-t|}\left(\Gamma\delta_{+1,\xi} - \Gamma\delta_{-1,\xi} + \Gamma_+ - \Gamma_-\right)\right]. \quad \text{(A2)}$$

The expectation values of all odd and even powers of $\xi(t)$ and its products can now readily be evaluated (since ξ is only allowed to assume the values ± 1). One finds

$$\begin{aligned} &\langle \xi^{2k}(t')\xi^{2n}(t)\rangle = 1 \\ &\langle \xi^{2k}(t')\xi^{2n+1}(t)\rangle = \langle \xi(t)\rangle = (\Gamma_- - \Gamma_+)/\Gamma \\ &\langle \xi^{2k+1}(t')\xi^{2n+1}(t)\rangle = \langle \xi(t')\xi(t)\rangle \equiv C(|t'-t|). \end{aligned} \quad \text{(A3)}$$

The function $C|(t'-t|)$ defined in equation (A3) can be calculated using its definition, which can be re-expressed as

$$C(t', t) = \sum_{\xi,\xi'=\pm 1} \xi'\xi\, Q(\xi', t'|\xi, t)P(\xi, t). \quad \text{(A4)}$$

$P(\xi, t)$ is here the one-event probability of finding the value ξ at time t. We find

$$C(t', t) = \frac{1}{\Gamma^2}\left[(\Gamma_+ - \Gamma_-)^2 + 4\Gamma_+\Gamma_- e^{-\Gamma|t'-t|}\right]. \quad \text{(A5)}$$

Expanding the exponential in the generating function (19) and making use of the expectation values (A3), together with (A5) we obtain

$$\begin{aligned} K(x, y|\tau) = \cos x \cos y - \sin x \sin y &\left[\frac{(\Gamma_+ - \Gamma_-)^2}{\Gamma^2} + \frac{4\Gamma_+\Gamma_-}{\Gamma^2} e^{-\Gamma|\tau|}\right] \\ &- \mathrm{i}\frac{\Gamma_+ - \Gamma_-}{\Gamma}\sin(x + y). \end{aligned} \quad \text{(A6)}$$

For the difference function $\mathcal{G}(x, y|\tau)$ defined in (19) we get

$$\mathcal{G}(x, y|\tau) = -\sin x \sin y \frac{4\Gamma_+\Gamma_-}{\Gamma^2} e^{-\Gamma|\tau|} = -\frac{\sin x \sin y}{\cosh^2(\Delta/2k_BT)} e^{-\Gamma|\tau|}. \quad \text{(A7)}$$

Here we have used the relation (13) between Γ_+ and Γ_-.

References

[1] Glazman L I and Khaetskii A V 1989 *Europhys. Lett.* **9** 263
[2] Kouwenhoven L P, van Wees B J, Harmans C J P M, Williamson J G, van Houten H, Beenakker C W J, Foxon C T and Harris J J 1989 *Phys. Rev.* B **39** 8040
[3] Zagoskin A M 1990 *Pis. Zh. Eksp. Teor. Fiz.* **52** 1043 (Engl. Transl. 1991 *JETP Lett.* **52** 435)
[4] Patel N K, Nichols J T, Martin-Moreno L, Pepper M, Frost J E F, Ritchie D A and Jones G A C 1991 *Phys. Rev.* B **44** 13 549
[5] Beenakker C W J and van Houten H 1991 *Solid State Physics* vol 44 (New York: Academic) p 1
[6] Taboryski R, Geim A K and Lindelof P E 1992 *Superlatt. Microstruct.* **12** 137
[7] Taboryski R, Geim A K, Persson M and Lindelof P E 1994 *Phys. Rev.* B **49** 7813
[8] Szafer A and Stone A D 1989 *Phys. Rev. Lett.* **62** 300
[9] Kirczenow G 1989 *Phys. Rev.* B **39** 10 452
[10] Zagoskin A M and Kulik I O 1990 *Fiz. Nizk. Temp.* **16** 911 (Engl. Transl. 1990 *Sov. J. Low. Temp. Phys.* **16** 533)
[11] Xu Hongqi 1993 *Phys. Rev.* B **47** 15 630
[12] Zagoskin A M and Shekhter R I 1994 *Phys. Rev.* B **50** 4909
[13] Laughton M J, Barker J R, Nixon J A and Davies J H 1991 *Phys. Rev.* B **44** 1150
[14] Glazman L I and Jonson M 1991 *Phys. Rev.* B **44** 3810
[15] Lindeløf P E private communication
[16] Glazman L I, Lesovik G B, Khmelnitskii D E, and Shekhter R I 1988 *Pis. Zh. Eksp. Teor. Fiz.* **48** 218 (Engl. Transl. 1988 *JETP Lett.* **48** 238)

[17] Brataas A and Chao K A 1993 *Mod. Phys. Lett.* B **7** 1021
[18] Galperin Yu M, Karpov V G and Kozub V I 1989 *Adv. Phys.* **38** 669
[19] The properties of a single activated EF are usually assumed to be independent of V_{sd} as well as gate voltage. Nevertheless, the *number* of activated EFs depends on both these voltages. Here we only study one EF, so this dependence does not enter.
[20] This assumption will probably be invalid for the experimental situation in [21]. Here rearrangements of atoms within a mechanically controllable break junction cause the current to vary in time. In this case one can expect a strong dependence of Γ on the bias voltage.
[21] Muller C J, van Ruitenbeck J M and de Jongh L J 1992 *Phys. Rev. Lett.* **69** 140
[22] Timp G L and Howard R E 1991 *Proc. IEEE* **79** 1188
[23] Timp G, Behringer R E and Cunningham J E 1990 *Phys. Rev.* B **42** 9259
[24] Dekker C, Scholten A J, Liefrink F, Eppenga R, van Houten H and Foxon C T 1991 *Phys. Rev. Lett.* **66** 2148
[25] Ralls K S, Skocpol W J, Jackel L D, Howard R E, Fetter L A, Epworth R W and Tennant D M 1984 *Phys. Rev. Lett.* **52** 228
[26] Rogers C T and Buhrman R A 1984 *Phys. Rev. Lett.* **53** 1272
[27] Ralls K S and Buhrman R A 1988 *Phys. Rev. Lett.* **60** 2434
[28] Zimmerman N M, Golding B and Haemmerle W H 1991 *Phys. Rev. Lett.* **67** 1322
[29] Golding B, Zimmerman N and Coppersmith S N 1992 *Phys. Rev. Lett.* **68** 998
[30] Ralph D C and Buhrman R A 1992 *Phys. Rev. Lett.* **69** 2118
[31] Anderson P W, Halperin B I and Varma C M 1972 *Phil. Mag.* **25** 1
[32] Phillips W A 1972 *J. Low Temp. Phys.* **7** 351
[33] Karpov V G, Klinger M I and Ignat'ev F N 1982 *Solid State Commun.* **44** 333; 1983 *Sov. Phys.–JETP* **57** 439
[34] Khlus V A 1987 *Sov. Phys.–JETP* **66** 1243
Lesovik G B 1989 *JETP Lett.* **49** 592
Büttiker M 1990 *Phys. Rev. Lett.* **65** 2901; 1991 *Physica* **175** 199; 1992 *Phys. Rev.* B **45** 3807; 1992 *Phys. Rev.* B **46** 12485
[35] Price J 1984 *Surf. Sci.* **143** 145
[36] Gardiner C W 1985 *Handbook of Stochastic Methods* (Berlin: Springer)